Human versus Machine Problem-Solving: Winning Openings in Dakon /

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Recently, a winning opening for the game of Dakon was found by hand. This sequence of moves leads to a direct win for the beginning player, without the opponent even giving the opportunity to move. In this paper we investigate how difficult it is to find such winning openings for the computer and how the search methods differ for man and machine.

Introduction

Dakon is a game from the large group of mancala games that is played in South-East Asia. The game is played on a board with two rows of holes and a store on both sides (see figure 1). The two players sit at either side of the board (South or North). Each player has the own store at the right hand. At the start of the game, all holes (except the stores) are filled with an equal number of counters (which can be stones, seeds or shells). The objective of the game is to collect so many counters in the own store that the opponent cannot move anymore in the next round.

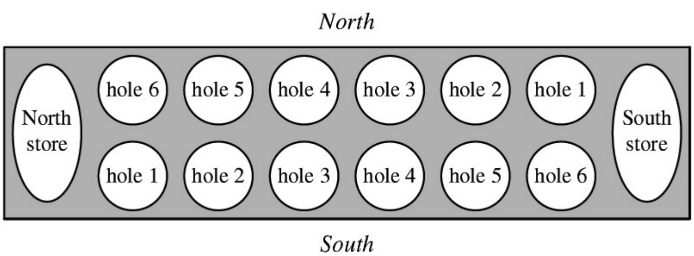

Figure 1: A Dakon board with 6 holes per side.

Dakon in the Maldives

Dakon as it is described by Murray (1952), Deledicq & Popova (1977) and Russ (2000) is known as Sungka in the Philippines, and as Conka/Congka/Congkak in Indonesia where it is also known as Dakon. The game was recently described for the Maldives (de Voogt 1999) where it is known as Ohvalhu. The winning opening was found by hand in the Maldives. A number of Maldivian women are familiar with this opening and learned the sequence by heart. There are no tournaments of the game in the Maldives and the game is mostly played in-house by women and children and sometimes men. During Ramadhan it is a popular pastime and not infrequently women and men play together.

There are four board-sizes known. Ohvalhu generally refers to a board of two rows of eight holes and two end-holes, but two rows of six, seven or even ten have also been documented. In those cases the number of counters by hole is adjusted to the number of holes in the row. There are a few variations of the game found and the rules most similar to Dakon are known throughout the islands.

Dakon is used instead of Ohvalhu since Ohvalhu is not a name generally known or used in the literature of mancala. Since the rules are similar to those found elsewhere in South-East Asia, a name from this area and used by all three sources mentioned above has been chosen for reasons of convenience.

Dakon rules

The rules for playing Dakon are as follows: The first round begins with agreeing who is going to start. The player at move selects one of the holes on the own side of the board that contains any counters. The player picks the counters out of the hole and puts ("sows") them one-by-one in the holes next to the starting hole, in anti-clockwise direction. The player's store also gets one counter, but the opponent's store is skipped. When the amount of counters in the selected hole is large enough, then the hole from which the sowing started will also receive a counter and the sowing continues until all counters are sown. This is called a lap. If the last counter of a lap is put in a hole that was not empty, all counters are taken out of that hole and sowing continues with the next hole. Such a sequence of laps will be called a move.

If the last counter of a move is put in the player's store, the player can select a new hole from the own side and move again. When the last counter of a move is put into an empty hole at the player's side, the counter in that hole and all counters in the opposite hole are captured and put in the player's store and the player's turn is over. When the last counter is put into an empty hole at the opponent's side, the turn is over without capture. A turn in Dakon thus consists of a sequence of moves that all but the last end in the player's store.

A round ends when one of the players cannot move anymore. All remaining counters at the own side of the players are collected into their stores. The counters are then redistributed as follows: from the left to the right, the holes are filled with the original number of counters. Remaining counters are put in the store. If the number of counters is too low to fill all holes, the holes that cannot be filled are marked with a leaf and are skipped in the next round. The player that won the last round starts playing. The game is over when at the end of a round one of the players cannot even fill one hole.

There are a number of variants of the game known. Sometimes Dakon is played by sowing in clockwise direction and having the store at the left-hand side. In the game of Sungka, as it is played in the Philippines, both players start simultaneously in the first round. The player that can move the longest starts the next round. The winning opening for Dakon (see below) was found by players of the Maldives where the game is known as Ohvalhu: they play in a clockwise direction.

We will use the notation Dakon-n to indicate the game of Dakon on a board with n holes on both sides and starting with n counters in every hole.

Winning openings for Dakon

The game of Dakon has a special property that might also be present in other Mancala games: winning openings. A winning opening for Dakon-n is a sequence of moves that form the first turn of the starting player and that captures at least $2n^2 - n$ counters. It is winning because the opponent will have less than n counters left and cannot fill a single hole in the next round, which means that the game is over. Of course, all moves of a winning opening but the last one must end in the player's store.

A hand-found winning opening

The winning opening for Dakon-8 that was found by players of the Maldives is:

1, 8, 8, 6, 4, 6, 2, 3, 7, 4, 5, 1, 8, 8, 3, 8, 4, 8, 7, 8, 5, 2, 7, 8, 6, 5, 6, 3, 7, 4, 5, 2, 5, 8, 8, 6, 8, 3, 8, 5, 8, 7, 4, 8, 7, 8, 7, 8, 8, 6, 8, 7, 8, 4, 8, 6, 8, 3, 8, 6, 8, 5, 8, 6, 1, 8, 7, 8, 5, 8, 4, 6, 7, 8, 8, 5, 6, 8, 3, 8, 1, 8, 7, 8, 2, 8, 6, 8, 5, 8, 6, 8, 3

The numbers indicate the holes that have to be selected at every move. Holes are numbered from 1 to 8 from left to right. This sequence consists of 93 moves and in total 168 laps. The move with the largest number of laps for a single move was 7 and occurred in move 77. After the complete sequence there are only three counters left in the opponent's holes. Figure 2 shows the first 5 moves of this winning opening.

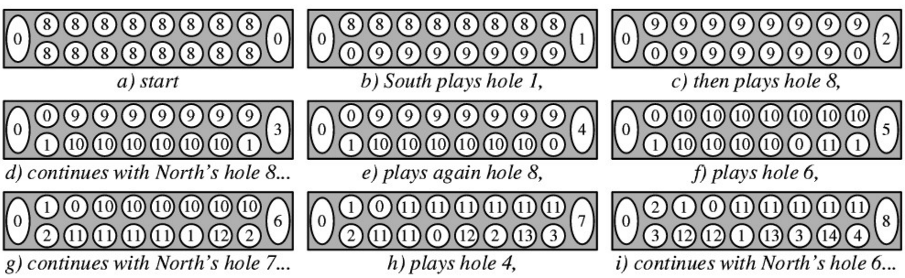

Figure 2: First 9 situations of the winning opening for Dakon-8.

Are such winning openings always possible? This question can only be answered by exhaustive search. We can however show that winning openings are not impossible. In Dakon-n, the minimum number of counters that will remain at the opponent's side after any opening sequence of moves is n divided by two, minus 1, the result being rounded to below (or as a formula: $n / 2 - 1$).

This can be proven as follows: a first observation is that during a move, two adjacent holes at the opponent's side of the board can never be empty at the same time, if the player is not actually sowing. An opponent's hole k can only become empty after a capture (which ends the turn) or when hole k is the end of a lap and contains more than one counter. In this case, hole k is the start of a new lap in the move. The previous hole ($k - 1$) must have at least one counter, otherwise the lap could never have ended in hole k. The next hole ($k + 1$) will contain at least one counter as soon as the seeds from hole k are sown. The fact that (before a capture) no adjacent opponent holes can be empty, means that the minimum number of counters that have to be present at the opponent's side is n / 2 , the minimum number of non-empty holes. At the end of the turn, at most one of these counters could be captured, so the minimum number of counters that will be left at the opponent's side is n / 2 - 1.

This result is no guarantee that a winning opening actually exists. It merely shows that a winning opening move could exist. Below we will show that there is no winning opening for Dakon-3.

Human problem-solving

Limitations of calculation

Psychological research on mancala players is limited to developmental psychological findings concerning Awele players (Retschitzki 1990) and cognitive psychological findings concerning Bao players (de Voogt 1995). The experiments conducted on Bao can be related to the expertise shown by the Maldivian players. If only two rows of Bao are used and moves without captures are calculated then the complexity of Bao is similar to that of Dakon. There remains a small difference in rules and Dakon uses twice as many counters on the board. The results from Bao indicate that independent from the number of holes per row, players are limited by the number of cycles they can calculate. A cycle is a move that travels the entire board once. For expert players this limitation is set to about four cycles while only in exceptional cases this may reach six.

In Dakon, cycles are commonly not calculated. If they are calculated they are limited to one or perhaps two cycles. The high number of counters would make it necessary to take counters from the board in order to count them, which is not allowed. Also, the high number of counters would allow for moves continuing for many cycles. For the above reason it is expected that a possible winning opening for the game of Dakon has a minimum number of moves that exceed one or two cycles. In such a case it is likely that the opening is found 'by hand'. As a consequence the likely opening to be found tends to be the longest rather than the shortest in terms of number of moves. In other words, the human search method minimizes the number of cycles per move while the computer search method could minimize the number of moves themselves. If all the moves that can be calculated, i.e., not continuing for more than one or two cycles, fail to continue the series, then a single move that cannot be calculated is easily identifiable as the next move. If during the 93 moves there occurs a situation on the board in which a choice between various multi-cycle moves is necessary, there is an extra mnemonic aid necessary to locate the next move. Similarly, if there are more than one single-cycle moves there is also an extra mnemonic aid necessary.

Chunking

In the cognitive psychological literature the concept of chunking is used to describe the memory process by which experts remember complicated positions or series of moves (Chase & Simon 1973, see also De Groot & Gobet 1996). The sowing series described for Dakon includes 93 moves. This series has been learned by heart by various players.

Bao, Chess and other players who remember series of moves use chunking. This means that they group moves together in larger units or chunks. Experiments of this kind involve a limited time-span in which the players were asked to remember series of moves. In longer time-spans this chunking technique can be perfected and other mnemonic techniques could be used. In the case of Dakon, it is expected that the moves are grouped together. Although there are many ways of chunking a group of moves, it is argued here that those moves are grouped together that use the same procedure for locating the next move. For instance, the first 10 moves form a group. Memory strategies for identifying a group are endless and the dedicated reader who plays the 93 moves and tries to remember them will immediately devise such strategies as well.

Solution finding

The limits of calculation and the chunking techniques may explain part of the memorization process but do not necessarily explain why this 93-moves winning opening was found by the players of the Maldives. It is explained that the problem-solving strategy is likely to minimize the number of cycles per move. This explains why the opening is not equal to the shortest winning opening found by computer (see below). Another solution finding approach is trial and error. The considerable number of possibilities of Dakon make this approach unlikely as an overall strategy. If the number of choices is small, i.e. the other choices can be calculated and found inadequate, the trial-and-error approach is more appropriate.

Winning openings, found by computer search

A computer is different from the human mind. Unlike humans, the computer excels in trying all possibilities in a systematic and painstaking manner. A computer can check all possible move sequences in order to find winning openings for Dakon. In our computer program, the move sequences are generated in a specific way by a procedure called backtracking. At every stage, the program checks which moves are possible and performs (in memory) the first move from the left. This results in a new board situation in which the program again finds the possible moves and tries the first one. The program continues until no moves can be done anymore. This can happen because the last move was a capture or because all holes on the player's side are empty. The program now checks whether enough counters are being captured for the sequence to be a winning opening. If so, the sequence is written down. If not, the last move is undone (which is the actual backtracking) and the next possible move is tried. If all moves at a stage are tried, the move of the previous stage is undone and the search continues. The process continues until all sequences are checked. Finding a winning opening is fast. Up to Dakon-18, winning openings are found within seconds on a standard desktop PC (300 Mhz Pentium II, the program has been written in the language Java).

It is possible to adjust the program so that it finds all winning opening. For Dakon-2 (which is obviously never played), there appear to be only 2 possible sequences of moves, and both sequences are winning openings. Dakon-3 happens to have no winning openings at all among the 13 possible move sequences. For Dakon with more than 3 holes, winning openings exist always, at least for up to 18 holes per side. Dakon-4 has 147 possible move sequences of which 7 are winning openings, Dakon-5 has 1,371 possible move sequences of which only 2 are winning openings and Dakon-6 has as many as 7,611,977 possible move sequences and 41,263 winning openings. For Dakon with more than 6 holes per side, the number of possible move sequences is too large to check. At least 50 million sequences where counted for Dakon-7, which took 3 full days of computation. If one looks at the growth rate of the number of possible sequences, the total time to check Dakon-7 might take as much as 6000 days on our computer system! This means that the exact number of winning opening moves for Dakon with more than six holes per side could not be determined by us.

Maximum-capture winning openings

Because the number of winning openings is so large, it is interesting to find special types of winning openings and compare these to the hand-found solution. The first type is a winning opening which captures as many counters as possible. Table 1 gives an overview of the results. It appears that for Dakon with more than 5 holes, winning openings exist that capture the theoretical maximum. For Dakon-4 and Dakon-5 all winning openings fail to capture the theoretical maximum number of counters.

Shortest winning openings

Memorizing a long winning opening is difficult, so it is also of interest to find those winning openings that consist of the least number of moves. To find these openings, the search procedure must be adapted slightly. Whenever a winning opening of a certain length has been found, all further move sequences that are at least as long can be skipped. This speeds up the search and enables us even to find the shortest winning opening for Dakon with 8 holes per side (see Table 2). Observe that this solution has only 29 moves, against the 93 moves of the hand-found solution. For Dakon-9 and Dakon-10, the time for completing the search took too long: it was aborted after 4 days of computing. The best results so far are marked with an asterisk in the table.

Human-like winning openings

A next question we can pose is whether the computer is able to find "human-like" winning openings. A first attempt is to find winning openings in which the maximum number of laps per move is as small as possible and, within this, the total number of moves is as small as possible. Table 3 (p. 88) shows our findings. It appears that as the number of holes per side increases, the lap count decreases and stabilizes at 4 or 5. Asterisks in the third column of table again indicate that some of the computations have been aborted and only best results found are given. The asterisks in the second column indicate that we could not find, within reasonable time, any winning openings for smaller lap counts.

The second attempt to find human-like winning openings is to minimize the number of cycles per move. This is done as follows: first the maximum number of cycles per

# Holes	# Counters left	# Solutions	Example solution
4	2	2	1, 4, 2, 4, 2, 2, 2, 4, 3, 1, 4, 4, 3, 4, 1
5	4	2	1, 5, 3, 1, 5, 2, 5, 5, 4, 3, 1, 3, 5, 5, 4, 3, 5, 5, 1, 4, 2, 5, 1, 5, 2, 5, 3
6	2	415	1, 5, 1, 3, 2, 3, 5, 2, 3, 5, 2, 3, 1, 3, 6, 2, 2, 5, 2, 2, 6, 4, 6, 5, 2, 5, 3, 6, 6, 2, 6, 3, 6, 5, 6, 1
7	2	Unknown	1, 7, 7, 5, 3, 5, 7, 7, 4, 3, 4, 5, 6, 3, 1, 5, 1, 7, 2, 5, 3, 7, 7, 4, 7, 1, 5, 3, 7, 7, 2, 4 ,7 ,7 ,6 7, 5, 7, 3, 2, 4, 1, 6, 4, 7, 7, 6, 3, 7, 3, 7, 5, 7, 6, 7, 1
8	3	Unknown	1, 8, 8, 6, 7, 8, 8, 4, 7, 6, 8, 5, 8, 8, 5, 5, 7, 1, 7, 2, 7, 4, 2, 8, 5, 6, 3, 8, 7, 6, 5, 8, 8, 3, 1, 7, 2, 7, 4, 5, 6, 8, 3, 7, 8, 5, 4, 8, 7, 8, 1, 4, 7, 8, 8, 6, 8, 1, 8, 3, 8, 5, 7
9	3	Unknown	1, 9, 9, 7, 9, 9, 4, 8, 9, 5, 7, 5, 5, 8, 9, 8, 7, 9, 6, 7, 9, 3, 6, 8, 9, 8, 7, 9, 9, 7, 9, 3, 6, 2, 8, 8, 3, 2, 7, 8, 8, 9, 8, 4, 6, 9, 3, 9, 5, 9, 7, 9, 9, 5, 3, 4, 9, 8, 2, 8, 4, 3, 9, 6, 9, 3, 9, 7, 9, 1, 9, 4, 9, 6, 9, 8, 9, 3
10	4	Unknown	1, 10, 10, 8, 10, 9, 10, 10, 7, 6, 10, 8, 10, 10, 10, 10, 7, 3, 8, 6, 10, 9, 7, 5, 6, 10, 8, 6, 7, 5, 6, 8, 9, 4, 1, 1, 6, 10, 7, 9, 5, 8, 3, 8, 5, 8, 7, 8, 4, 6, 1, 4, 5, 2, 1, 7, 6, 8, 1, 8, 6, 5, 8, 10, 9, 10, 6, 10, 5, 10, 10, 8, 10, 7, 10, 9, 8, 9, 10, 8, 10, 1, 10, 8, 10, 7, 10, 5, 10, 9, 10, 3, 10, 5, 10, 4, 10, 5, 10, 9, 10, 7

Table 1: Maximum-capture winning openings.

# Holes	Length	Example solution
4	12	1, 4, 2, 4, 2, 2, 2, 4, 3, 1, 4, 4, 3, 4, 1
5	24	1, 5, 3, 1, 5, 2, 5, 5, 4, 3, 1, 3, 5, 5, 4, 3, 5, 5, 1, 4, 2, 5, 1, 5, 2, 5, 3
6	19	1, 5, 2, 6, 6, 1, 4, 6, 2, 6, 3, 2, 6, 6, 4, 6, 5, 6, 3
7	25	1, 7, 4, 1, 6, 7, 4, 3, 1, 1, 4, 1, 4, 7, 2, 1, 5, 2, 3, 6, 1, 5, 5, 7, 1
8	29	1, 4, 4, 5, 3, 8, 4, 2, 1, 5, 5, 4, 6, 5, 5, 2, 3, 6, 4, 7, 6, 7, 8, 8, 8, 5, 8, 8, 6
9	41*	1, 9, 9, 7, 9, 9, 4, 8, 9, 5, 7, 5, 5, 4, 3, 8, 8, 6, 8, 1, 1, 7, 9, 9, 6, 5, 7, 9, 9, 9, 6, 5, 7, 9, 8, 5, 4, 9, 8, 9, 7
10	68*	1, 10, 10, 8, 10, 9, 10, 10, 7, 6, 10, 8, 10, 10, 10, 10, 7, 3, 8, 6, 10, 9, 7, 5, 6, 10, 8, 6, 7, 5, 6, 8, 9, 4, 1, 1, 6, 10, 5, 9, 3, 7, 10, 5, 9, 6, 9, 4, 10, 7, 10, 7, 7, 5, 10, 4, 10, 6, 10, 1, 7, 8, 8, 10, 4, 8, 10, 10, 8

Table 2: Shortest winning openings. (* shortest found so far)

# Holes	Lap Count	Length	Example solution
4	8	13	1, 4, 2, 4, 2, 2, 2, 4, 3, 4, 1, 4, 3
5	6	27	1, 5, 3, 1, 5, 2, 5, 5, 4, 3, 1, 3, 5, 5, 4, 3, 5, 5, 1, 4, 2, 5, 1, 5, 2, 5, 3
6	5	31	1, 6, 4, 2, ,4 ,3, 3, 4, 5, 3, 5, 2, 1, 5, 2, 5, 2, 4, 5, 2, 5, 6, 2, 5, 4, 6, 1, 6, 6, 3, 4
7	5	49	1, 7, 7, 5, 3, 5, 7, 5, 6, 7, 7, 3, 1, 3, 6, 5, 6, 4, 1, 2, 2, 5, 4, 6, 2, 7, 6, 7, 3, 7, 6, 1, 6, 7, 1, 7, 6, 7, 5, 6, 7, 5, 7, 3, 1, 6, 7, 1, 7
8	4	75*	1, 8, 8, 6, 4, 6, 2, 3, 7, 6, 7, 8, 8, 5, 3, 6, 2, 4, 7, 6, 5, 8, 8, 3, 8, 5, 2, 4, 8, 1, 5, 4, 2, 5, 4, 7, 5, 8, 2, 7, 8, 4, 8, 7, 8, 3, 8, 5, 6, 7, 3, 8, 8, 1, 8, 5, 8, 4, 7, 8, 5, 1, 8, 8, 2, 8, 4, 8, 7, 8, 5, 8, 6, 8, 1
9	5*	103*	1, 5, 1, 4, 6, 4, 5, 3, 1, 4, 2, 1, 3, 9, 6, 4, 8, 4, 7, 2, 3, 9, 9, 5, 9, 8, 7, 4, 9, 7, 9, 7, 4, 5, 3, 9, 8, 2, 4, 5, 6, 7, 1, 6, 3, 7, 5, 2, 8, 2, 7, 6, 9, 3, 1, 9, 9, 8, 8, 6, 9, 6, 7, 7, 4, 9, 8, 9, 6, 9, 7, 8, 2, 5, 9, 1, 9, 9, 8, 9, 6, 9, 4, 9, 8, 9, 1, 9, 6, 9, 5, 9, 6, 9, 8, 9, 3, 9, 4, 9, 5, 9, 6
10	5*	129*	1, 6, 1, 7, 2, 3, 1, 6, 4, 4, 2, 9, 2, 4, 8, 3, 4, 8, 1, 10, 5, 10, 6, 9, 7, 1, 8, 1, 1, 6, 7, 8, 5, 1, 7, 5, 10, 7, 1, 2, 9, 8, 4, 9, 1, 2, 7, 8, 2, 5, 9, 10, 7, 3, 1, 10, 7, 10, 9, 2, 4, 3, 9, 8, 7, 9, 8, 9, 6, 5, 6, 1, 7, 6, 5, 9, 7, 10, 10, 8, 1, 10, 5, 8, 10, 1, 6, 5, 10, 10, 4, 5, 9, 8, 10, 10, 9, 4, 1, 10, 8, 7, 10, 10, 6, 9, 10, 10, 8, 10, 5, 10, 1, 10, 7, 10, 2, 10, 3, 10, 5, 10, 9, 10, 6, 10, 8, 10, 4

Table 3: Minimum-lap-count winning openings. (* best found so far)

move (measured in the number of counters involved in the move) is minimized and within this set of solutions, the number of moves that exceed 1 cycle is minimized. For Dakon-8, it appears that all solutions have a move that at least involves 39 counters (about 2.4 cycles). The best winning opening for Dakon-8 counts 86 moves among which only 22 exceed one cycle:

> 1, 8, 8, 6, 4, 6, 2, 1, 7, 5, 6, 8, 8, 2, 8, 4, 8, 1, 8, 7, 7, 1, 4, 6, 2, 6, 5, 4, 3, 2, 8, 8, 7, 6, 2, 1, 3, 6, 8, 4, 6, 3, 8, 6, 3, 8, 5, 8, 4, 8, 6, 6, 3, 8, 5, 4, 1, 8, 7, 8, 2, 4, 8, 8, 5, 8, 7, 8, 7, 8, 5, 8, 1, 8, 5, 3, 2, 8, 7, 8, 3, 8, 4, 8, 5, 7

Observe that the start of this winning opening equals the hand-found winning opening.

Analysis of the hand-found winning opening

The hand-found winning opening appears to have three characteristics. The number of cycles of each move is low with only one move which exceeds three cycles. The number of moves with more than one cycle is 22. The best winning opening found by computer has only 13 such moves but they frequently exceed three cycles. The best winning opening with the most single-cycle moves and the lowest maximum cycle appears 86 moves long and has 22 moves with go beyond one cycle. This predicts with some accuracy the hand-found sequence also because the start of the move is identical.

An analysis of each move in the hand-found sequence shows that moves 10, 13, 34, 42, 46, 48, 65-67 and 77 has more than one option that could be successful and minimize the number of cycles. They do not adhere to the minimal cycle requirement, while all other moves are optimal according to the above analysis.

Although this explains why this winning opening was found instead of others, it does not explain how the players could have analyzed so many options. The first 10 moves show a clear regularity when played. The last 10 moves are most likely necessary moves, meaning that any other option would lead to an immediate halt to the sequence. The middle moves can only be explained if the number of options that continue the sequence is limited. In that case the trial-and-error method in combination with cycle calculations predict a successful search within a reasonable time-span. It appears that when the number of possible continuations is high (more than 5) there is a reasonable chance that a less optimal choice is taken in the hand-found solution. Move 10, 13, 34, 66 and 67 were taken from a selection of six possible continuations. Move 46 and 48 were taken from eight possible continuations, indeed the maximum number of choices and the only two times that these maximums occurred in the hand-found opening.

Conclusions

There are solutions to Dakon games of sizes 2x4 to 2x10, sizes that are also known in the mancala literature. The small number of winning openings in relation to the total number of possible move sequences does not make a hand-found solution likely. The winning opening that was found in the Maldives appears to have particular characteristics that explain the problem-solving method of the players. It appears that the calculation ability of the players influences the type of solution they have found. Also, regular initial sequences and necessary end sequences determine a large portion of the moves they played. Although the characteristics of the solution can be explained, the accomplishment itself can only be laudated as an act of mastership comparable to the expertise shown in other board games of this kind.

It is shown that computer science and cognition in general has much to gain from games analyses such as these. The present study exemplifies the results that could be gained from such studies and should encourage further research of this kind.

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